

## Analysis script of the "Falling Cylinder " experiment

### A) Introduction to the experiment

All objects in free fall in the vicinity of the Earth's surface have a constant acceleration,  $g$ , when air resistance can be ignored. This experiment tests the validity of this property for a rigid body that falls, but has a constraint with an object that does not move out of place, which induces other movement in addition to translation.

### B) Analysis procedure

**B1. Model.** Watch the videos of the experiment and observe the movements of the cylinder and the tape as a whole. Reflect and **Write it down, using your own words** (estas anotações não serão pontuadas, mas você precisa anotá-las a fim de poder verificar sua concepção inicial quando terminar o relatório):

- i. How do you describe the movement of the cylinder? And the tape?
- ii. In the experimental arrangement, locate the cylinder and tape links. How are they maintained throughout the movement? Regarding the elements of the experimental arrangement that constitute these constraints, identify which of their properties are important to keep them intact throughout the movement.
- iii. What role do links play in cylinder movement? And in the one on the tape?
- iv. Does the above analysis suggest that the acceleration of the cylinder's center of mass is constant? And would its value be **less, equal to or greater** than  $g$ ?
- v. Build a physical model of the experimental arrangement, that is, write a text that explains how the forces acting on the cylinder (and the tape, perhaps) cause the observed movement, without using equations, but including terms such as force, torque, moment of inertia, angular acceleration, etc.

**B2. Data taking.** Look at the images of the set that was assigned to you. The checkered reference board has black horizontal and vertical lines in which the thick ones are separated by 1 cm and the thin ones by 0.5 cm; has a red line every 5 cm.

Choose a point in the grid as the origin of the Cartesian coordinate system – it will be easier to choose the origin of the coordinate system on a red line. Adopt the center of the cylinder or the upper or lower end of the edge as a reference to measure its positions.

Set up a table, associating to each frame  $i$  the instant of time  $t_i$  and the vertical position  $y(t_i)$  of the cylinder into the grid. Adopt 2 mm for the standard deviation of the positions, that is,  $\sigma_y = 0,2 \text{ cm}$ , and ignore the uncertainty in time.

**B3. Cinematic Quantities.** Calculate the average velocity of the cylinder center in *the laboratory frame of reference*, for the time intervals  $[t_{i-1}; t_{i+1}]$ , and use that value as an estimate of instantaneous velocity at the instant  $t_i$ , as:

$$v(t_i) \cong \bar{v}(t_i) = \frac{y(t_{i+1}) - y(t_{i-1})}{t_{i+1} - t_{i-1}} \quad (1)$$

where  $y$  and  $t$  correspond, respectively, to the vertical position and the instant of time, while  $\bar{v}$  symbolizes the linear mean velocity and  $i$  the number of the frame in question.

Also calculate the standard deviation in velocity from the equation below:

$$\sigma_{v_y} = \frac{\sqrt{2}}{|t_{i+1} - t_{i-1}|} \sigma_y \quad (2)$$

**B4. Experimental result.** Construct the graph of the cylinder velocity as a function of the time calculated in item **B3** and adjust a trend line to this graph. Also insert the uncertainty bars. Identify and interpret the slope adjusted to the data by the spreadsheet,  $a$ . According to the data processing guide (on the auxiliary tabs, document [Estatística: MEXI](#)), compute the standard deviation of the slope  $a$  as:

$$\sigma_a = \frac{\sigma_{v_y}}{T} \sqrt{\frac{12}{N}} \quad (3)$$

where  $T$  is the total time interval during which the vertical velocity of the cylinder was measured, and  $N$  is the number of data.

**B5. Mathematical model.** Build a mathematical representation of the system that allows you to determine the acceleration of the cylinder and find a formula for this quantity from the properties of the elements of the arrangement. To do so, solve the exercise below:

A tape is wound on the surface of a hollow cylinder, of mass  $M$ , with outer and inner radii equal to  $R$  and  $r$ , respectively. The tape has a fixed end and the cylinder falls vertically, as in the video of the experiment. To determine your acceleration during the fall, follow the steps:

- Make a sketch of the arrangement and the free-body diagram of the object under study.
- Write down the equation of motion (or the equations of motion, depending on which path you choose).
- Find formulas that relate the physical properties of the elements of the arrangement (radius, mass, etc.) to the parameters of the equation (or equations) of item b).
- Substitute these formulas into the equation (or equations) of motion of item b).
- Algebraically isolate the linear acceleration of the cylinder as a function of  $g$  and the radius(es).

**B6. Calculated acceleration.** For each of the possible pairs of inner and outer radii of the cylinders used in this experiment, listed in the materials tab, numerically calculate the acceleration predicted by the mathematical model. Do this calculation on a spreadsheet, so that you can use the same formula for all cylinders.

**B7. Uncertainty in acceleration.** Calculate the standard deviation of the value calculated in **B6**, as a function of the standard deviations of the cylinder's inner and outer radii. As in calculating accelerations, do the calculations in the spreadsheet to use the same formula for all cases.

**B8. Similarity.** Identify the cylinder that starred in the set of images you analyzed and comment if there is ambiguity in the result. Determine the degree of similarity between the experimental arrangement and the model you developed, assuming that you have correctly identified the cylinder used.

## Report preparation procedure

Each group must submit a single document, with the following sections:

**C1. Identification.** List the names of the group members and indicate the analyzed dataset.

**C2. Experiment Description.** Describe what you observe in the video and the components of the experimental arrangement succinctly. Include all the elements that play a role in the movement as you see it in the images, making it clear how the objects are linked together in a way that results in the observed movement, but don't use formulas. The focus should be on the elements and their characteristics, on everything that is important to build a similar experimental arrangement. Write two versions of your description, one without using the names of physical quantities such as force, torque, inertia, as you would do for those who do not study physics, and another, using these names, but without using formulas, as you had to **think of** to write the equations necessary to calculate acceleration. To help, use your initial notes from item **B1**.

**C3. Mathematical model.** Develop the mathematical representation of the model and present the expression for the moment of inertia of the cylinder as well as the formula for the acceleration obtained. Make it clear whether you considered the movement a pure rotation or a combination of rotation and translation and justify your choice. Discuss the dynamics of motion: Does the model indicate that the acceleration is constant or variable? Less, equal or greater than  $g$ ? Is the amount of movement constant or does it vary? Is the energy constant or does it vary? Explain, using only words, why the mass of the cylinder does not enter the expression of angular acceleration.

**C4. Data Obtained.** Present the tables obtained in items **B2** and **B3**. Check that you have expressed the values of the quantities in appropriate units and with an appropriate number of significant digits. Do not forget to mention how you performed the calculations and arrived at the results.

**C5. Data analysis.** Present the graph of the cylinder velocity as a function of time and the slope coefficient of the trend line (item **B4**). Present the table of expected accelerations for the different cylinders, accompanied by the respective standard deviations.

**C6. Results and discussion.** Point out which cylinder must have been used to generate the set of images you analyzed and evaluate the degree of certainty of your choice. Discuss whether the model is compatible with observation within the experimental uncertainties.

Comment on your initial expectation about the value of the cylinder acceleration obtained in item **B1**, and if it is consistent with the results obtained in **B4**. What properties of the system determine that the acceleration is (constant, variable) and (less, equal, greater) than  $g$ ? What was the role of the constraints in this result?

**C7. Conclusion.** Report the degree of success of the model in explaining the movement: resume the introduction, pay attention to the objective of the experiment and comment on whether it was fully achieved, partially or not, and why. (you can resume your hypotheses and questions recorded in the item **B1** and verify whether the experiment corroborated such initial expectations). Report whether the constraints interfered with the movement and whether this conclusion could be extended to any system. Conclude whether the accuracy of the experiment was sufficient to define which cylinder was used.